What you'll Learn About

- Average Rates of Change
- A Definition of the Derivative

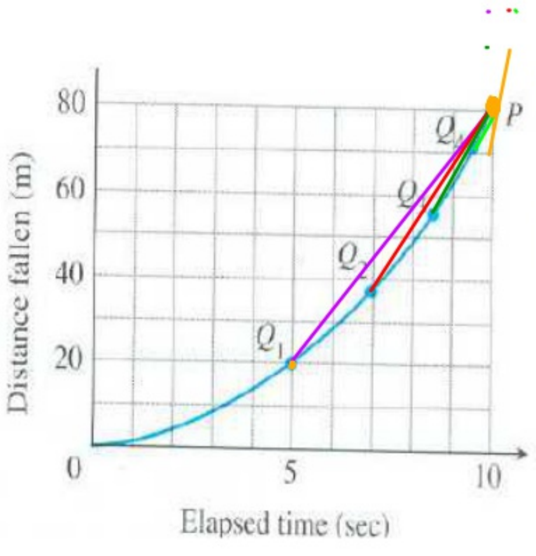
An object dropped from rest from the top of a tall building falls $y = 16t^2$ feet in the first t seconds. Find the **average speed/average rate of change** during the first 2 seconds of flight.

Find the average rate of change of $f(x) = \sqrt{4x+1}$ over each interval

a) [0, 2]

b) [10, 12]

Estimate the average rate of change by finding the slopes of each secant line. Indicate units of measure



Q1(5,20)
Q2(7,38)
Q3(8.5, 55)
Q4(9.5,71)
P(10,80)

PQ1= $\frac{80-20}{10-5} = \frac{60}{5} = 12$

PQ2= $\frac{80-38}{10-7} = \frac{42}{3} = 14$

PQ3= $\frac{80-55}{10-8.5} = \frac{25}{1.5} = 16\frac{2}{3}$

PQ4= $\frac{80-71}{10-9.5} = \frac{9}{.5} = 18$

$\frac{f(9)-f(10)}{9-10}$

Use the slopes of the secant lines to Estimate the instantaneous rate of change/slope at point P ← tangent line

Instantaneous rate of change = $\lim_{x \to 10} \frac{\Delta Y}{\Delta X} = 20 \text{ m/s}$

$\boxed{\lim_{x \to a} \frac{f(x)-f(a)}{x-a}}$

Limit of the slopes of the secant lines

Instantaneous Rate of Change

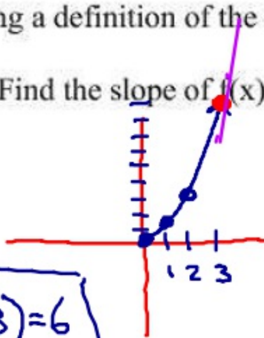
$a = 3$ (Pt of tangency)

Slope at a point Derivative $\lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a}$ $\boxed{\text{slope at } x=3 \text{ is } 6}$ $\boxed{\text{Derivative} = \text{slope} = f'(x)}$	Using a definition of the derivative to find slope

$f(x) = x^2$

$f(3) = 9$

A) Find the slope of $f(x) = x^2$ at the point (3,9)



$\boxed{f'(3) = 6}$

$\lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a}$

$\lim\limits_{x \to 3} \frac{f(x)-f(3)}{x-3}$

$\lim\limits_{x \to 3} \frac{x^2-9}{x-3} = \frac{0}{0}$

$\lim\limits_{x \to 3} \frac{(x+3)\cancel{(x-3)}}{\cancel{(x-3)}} = 6$

B) Find the slope of $f(x) = \frac{2}{x}$ at $x = 4$

$\lim\limits_{x \to a} \frac{f(x)-f(a)}{x-a}$

$\lim\limits_{x \to 4} \frac{\frac{(2)}{(2)}\frac{2}{x} - \frac{1}{2}\frac{(x)}{(x)}}{x-4} = \frac{\left(\frac{4-x}{2x}\right)}{x-4} = \frac{\cancel{(4-x)}}{2x} \cdot \frac{-1}{\cancel{(x-4)}} = \frac{-1}{2x} = -\frac{1}{8}$

$\boxed{f'(4) = -\frac{1}{8}}$

C) Find the slope of $f(x) = \frac{1}{x-4}$ at $x = 7$

$f'(7) = \lim\limits_{x \to 7} \frac{\frac{(3)}{(3)}\frac{1}{(x-4)} - \frac{1}{3}\frac{(x-4)}{(x-4)}}{x-7} = \frac{\frac{3 - 1x + 4}{3(x-4)}}{(x-7)} = \frac{\left(\frac{7-x}{3(x-4)}\right)}{(x-7)}$

$\frac{7-x}{3(x-4)} \cdot \frac{1}{x-7} = \frac{-1}{3(x-4)}$

$f'(7) = -\frac{1}{9}$

D) Find the slope of $f(x) = 9 - x^2$ at the point (-3,0)

$\lim\limits_{x \to -3} \frac{9-x^2-(0)}{x-(-3)} = \frac{9-x^2}{x+3} = \frac{(3-x)(3+x)}{x+3} = 6$

$f'(-3) = 6$